Cluster Programme - M.Tech. in Data Science and Engg.

II Semester 2018-19

Course Number	DSECF ZC416
Course Name	Mathematical Foundation for Data Science
Nature of Exam	Open Book
Weightage for grading	40%
Duration	2 hours and 30 minutes
Date of Exam	28/09/2019 (10:00 - 12:30)

# Pages	2
# Questions	4

Instructions

1. All questions are compulsory
2. Questions are to be answered in the order in which they appear in this paper and in the page numbers mentioned before each of them.

Pages 2-6

Q1a) Prove or disprove: If S is a finite subset of a vector space V, then the linear span $L(S)$ is the smallest subspace of V containing S. (2)

b) Prove or disprove: There is a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ with $\text{Ker}T = \{0\}$. (2)

c) Prove or disprove: The eigenvectors of $\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$ are $\begin{bmatrix} i \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -i \\ 1 \end{bmatrix}$ for $\theta \neq 0, \pi$ (2)

d) Apply Gauss elimination with partial pivoting to solve (4)

$$\begin{pmatrix} 0.02 & 0.01 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 100 & 200 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0.02 \\ 1 \\ 4 \\ 800 \end{pmatrix}$$

Pages 7-11

Q2a) Prove or disprove $\lfloor x \rfloor + \lfloor y \rfloor + \lfloor x + y \rfloor \leq \lfloor 2x \rfloor + \lfloor 2y \rfloor$ for all real numbers x and y. (2)

b) Draw the graph of the function $f(x) = \left\lfloor 2 \left\lceil \frac{x}{2} \right\rceil + \frac{1}{2} \right\rfloor$ (2)

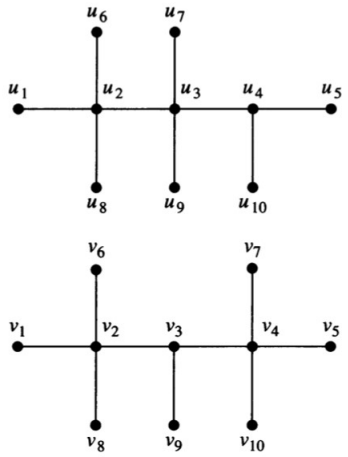
c) Use Warshall's algorithm to find the transitive closure on $\{a, b, c, d, e\}$ for the relation $\{(a, b), (a, c), (a, e), (b, a), (b, c), (c, a), (c, b), (d, a), (e, d)\}$. (2)

d) A relation R is called *circular* if aRb and bRc imply that cRa. Show that R is reflexive and circular if and only if it is an equivalence relation. (4)

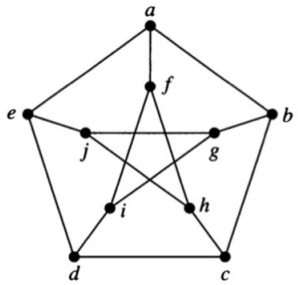
Pages 12-15

Q3a) Find the number of paths of length 3 between any two adjacent vertices in $K_{3,3}$ (3)

b) Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that it is not isomorphic. (3)



c) Show that the graph shown below does not have a Hamilton circuit, but that the subgraph obtained by deleting a vertex and all edges incident with it does have a Hamilton circuit. (4)



Pages 16-21

Q4a) Construct the ordered rooted tree whose preorder traversal is

$$a, b, f, c, g, h, i, d, e, j, k, l$$

where a has four children, c has 3 children, j has two children, b and e have one child each, and all other vertices are leaves. (3)

b) Represent $(A \cap B) - (A \cup (B - A))$ using an ordered rooted tree. Write this expression in prefix, postfix and infix notation. (3)

c) Use Prim's and Kruskal's algorithm to find the minimum spanning tree for the weighted graph given below. (4)